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PREDICTING NONLINEAR RANDOM RESPONSE  
(INCLUDING SNAP-THROUGH) OF BUCKLED PLATES  
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## **DESIGN GUIDE FOR PREDICTING NONLINEAR RANDOM RESPONSE (INCLUDING SNAP-THROUGH) OF BUCKLED PLATES**

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# **DESIGN GUIDE FOR PREDICTING NONLINEAR RANDOM RESPONSE (INCLUDING SNAP-THROUGH) OF BUCKLED PLATES**

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## **SUMMARY**

This design guide describes a method for predicting the random response of flat and curved plates which is based on theoretical analyses and experimental results. The plate curvature can be due to postbuckling in-plane mechanical or thermal stresses. Based on a single mode formula, r.m.s. values of the strain response to broadband excitation are evaluated for different static buckled configurations using the equivalent linearization technique. The effects on the overall strain response due to instability motion of snap-through are included. Panel parameters include clamped and simply-supported boundaries, aspect ratio, thickness and length. Analytical results are compared with experimental results from tests with 12 in. x 15 in. aluminum plates under thermal loading in a progressive wave facility. Comparisons are also made with results from tests with a 2 in. x 15 in. x 0.032 in. aluminum beam under base mechanical excitation. The comparisons help to assess the accuracy of the theory and the conditions under which deviations from the theory due to effects of imperfection and higher modes are significant.

## **INTRODUCTION**

Although much work has been done recently on developing analytical methods for predicting nonlinear random response of structures (ref. 1-4), most of these procedures are too tedious for practical application and have not been experimentally validated. Therefore, it is necessary to rely on empirical formulae (refs. 5-6) and experience to design aircraft structures against sonic fatigue. Furthermore, the estimation of nonlinear responses in plate structures (especially

curved plates) is so uncertain that designers often choose to avoid nonlinear problems by making thicker and heavier structures. The resulting increase in weight may be unacceptable in many future classes of high performance aircraft. On the other hand, it should be noted that for flat and curved plates, nonlinear response can result in hardening-spring effects (increase of stiffness with amplitude of deformation) and thus offer weight saving if the dynamic deflection can be controlled.

Some of the reasons for the lack of validation of prediction methods for nonlinear random vibration due to acoustic loads are the difficulties in generating large enough forces with desired spectra and difficulties in maintaining consistent rigid boundaries of plate specimens without premature fatigue problems. The availability of high intensity acoustic test facilities is limited and most acoustic tests have been performed on built up structures (ref. 5-9). Although these tests have recorded large amounts of structural response and fatigue life data, no vigorous attempts have been made to compare these data with theoretical predictions. There is also a lack of experiments on nonlinear response of curved plates. Design guidelines have been suggested but these are only applicable to structures which are similar to the test specimens and in general neglect effects of nonlinear response.

The status of nonlinear random response research is that there is a significant amount of purely theoretical analyses and some practical experimental work but little or no interaction between the two. To bridge this gap, simplifications in the procedures on both sides are required. The compromising solution and the approach taken in this reported activity is to derive simple and approximate analytical prediction techniques and improve these through comparison with experiments on simple structures, such as, beams and single bay plate specimens.

A Rayleigh-Ritz analysis using one mode for representing the transverse displacement of a plate was developed to predict the buckling displacement due to

in-plane stresses and the resulting equation of motion. The nonlinear acoustic response including the mean and r.m.s. strain response are predicted by the equivalent linearization technique. Previous work (ref. 10) using this technique has been shown to give good correlation with experiment. The prediction procedure is capable of handling hardening and softening spring effects and snap-through motion from one static position to another. Snap-through motion has significant effects on a buckled plate and thus it is necessary to evaluate the ability of the sound pressure level to initiate snap-through motion and to evaluate overall r.m.s. strain response with frequent snap-through. Design charts have been compiled and are presented in this report. Comparison of analytical and experimental results have been made with data from acoustic tests on a thermally buckled plate and from a mechanical test with a buckled beam.

### SYMBOLS AND ABBREVIATIONS

a	width of the plate, in.
b	length of the plate, in.
f	frequency, Hz
$f_0$	fundamental resonance frequency
h	acoustic pressure, lb/in <sup>2</sup>
K	linear modal stiffness of the flat plate, lb./in <sup>2</sup>
$K_N$	nonlinear modal force coefficient, lb./in <sup>3</sup>
p	nondimensional force parameter, = $P/KQ_N$
P	externally applied modal force $P = \int_0^a \int_0^b h \phi(x, y) dy dx$ $P = KQ + K_N Q^3$ for a flat plate
q	nondimensional displacement coefficient = $Q/Q_N$
$q_0$	value of q for static buckling
Q	modal displacement (see definition w below)

$Q_N$	value of $Q$ for which linear restoring equals the nonlinear restoring force i.e. $KQ_N = K_N Q_N^3$ . Note also $Q=Q_N$ when $R=1$
$R$	$u/u_c - 1$ (the buckling parameter)
$s$	aspect ratio = $b/a$
$S_{hh}(f)$	spectral density of $h$ at frequency $f$ , dB/Hz
$S_{pp}(f)$	spectral density of $p$ at frequency $f$ , $\text{Hz}^{-1}$
$S_{pp}(f_o)_L$ or $S_L$	value of $S_{pp}(f)$ when r.m.s. response of $q$ is 1, assuming no nonlinear restoring force $S_{pp}(f_o)_L = 4\zeta/\pi f_o$
$S_s$	value of $S_{hh}(f)$ when snap-through motion starts, dB/Hz
$S_c$	value of $S_{hh}(f)$ when continuous snap-through motion starts, dB/Hz
$t$	thickness of the plate, in.
$u$	inplane edge shortening displacement, in.
$u_c$	value of $u$ at which buckling starts, in.
$w(x,y)$	transverse displacement of the plate at coordinate $(x,y)$ , in. $w(x,y) = Q \phi(x,y)$
$x,y$	coordinate system with $x$ -axis and $y$ -axis parallel to shorter side, $a$ , and longer side, $b$ , respectively
$\alpha = \frac{S_{pp}(f)}{S_{pp}(f_o)_L}$	nondimensional excitation parameter
$\Omega$	linear circular frequency of the flat plate, radians
$\zeta$	modal damping coefficient
$\rho$	density of the plate material, $\text{lbm/in}^3$
$\gamma$	in-plane stiffness parameter
$\epsilon$	surface strain in microstrain due to bending = $\frac{t}{2} \frac{\partial^2 w}{\partial x^2}$
$\epsilon_o$	value of $\epsilon$ for static buckling
$\epsilon_N$	value of $\epsilon$ corresponds to $Q_N$
$\phi(x,y)$	shape function of the fundamental mode,

$\sin \frac{\pi x}{a} \sin \frac{\pi y}{b}$  for simply supported boundaries,

$\Psi(x/a) \Psi(y/b)$  for clamped boundaries

where  $\Psi(z) = 5/8 [\cosh dz - \cos dz - 0.98 (\sinh dz - \sin dz)]$ ,  $d = 4.73$

Note that maximum  $\phi(x,y)$  is 1

## **ANALYTICAL MODEL FOR SNAP-THROUGH MOTION**

Most previous work (ref. 11, 12) on theoretical prediction of snap-through under random excitation was concerned only with the stability boundaries but not with the magnitude of the nonlinear response. Also, the methods developed are lengthy and not easily applied to practical case of plates under acoustic excitation.

From reference 13, it is found that under low level random excitation a plate exhibits snap-through motion occasionally but the oscillation is generally around the initial static position. This is defined as the intermittent snap-through. Continuous snap-through motion is defined for higher levels of random excitation when snap-through motion constitutes more than 80 percent of the response. This usually occurs when the r.m.s. value of the dynamic displacement is equal to the initial static displacement. The general characteristics of the three regions of snap-through motions are summarized in Table 1.

The division of snap-through motion into three regions helps to understand the instability motions of buckled plates. The boundaries between them, however, are sometimes difficult to define, especially for shallow buckled plates, due to the small gap between the two static positions, or for buckled plates which have only one static position.

Snap-through boundaries were estimated empirically in reference 14 from a set of experimental results but no general formulae were derived. A formula for buckled plate behavior and an analysis model for the random response based on the above three types of snap-through motion was established in reference 10 and will be discussed next.

### General Equations for Nonlinear Behavior of Plates

Assuming an appropriate shape function for the transverse displacement of a plate and using the Rayleigh-Ritz procedure of formulation, the nonlinear equation for equilibrium in the transverse direction is obtained. The detailed steps in deriving the following equations are found in reference 10.

For a plate under uniaxial compression with uniform edge displacement, the nondimensional relationship between modal displacement,  $q$ , and modal force,  $p$ , for the buckling mode is given as:

for static equilibrium

$$q^3 - Rq = p \quad (1a)$$

and for dynamic motion

$$\ddot{q}/\Omega^2 + 2\zeta\dot{q}/\Omega + q^3 - Rq = p \quad (1b)$$

where  $R$  is the buckling parameter given by  $u/u_c - 1$ ,  $u$  is the in-plane edge displacement,  $u_c$  is the critical value for initiation of buckling,  $\Omega$  is the frequency of the plate and  $\zeta$  is the modal damping coefficient. The above equations are applicable to any plate of any boundary condition and aspect ratio if there is no coupling between the various modes. A plot of  $p$  versus  $q$  for various values of  $R$  from (1a) is shown in figure 1. From figure 1 regions of hardening and softening spring behavior and also regions of negative stiffness, (e.g. between A and B) can be found. Notice that for  $R=1$ , dynamic motion starting from C will pass through A and B and end up at C. This motion through the instability region is called snap-through. A and B are found to be at  $(0.577, -0.385)$ ,  $(-0.577, 0.385)$  respectively by locating the points where  $dP/dq = 0$ .



Static equilibrium positions  $q_0$  for positive values of the buckling parameter  $R$  are found by putting  $p=0$  in equation (1a) and correspond to the points where the curve crosses the  $q$  axis in figure 1,  $q_0 = \sqrt{R}$ ,  $-\sqrt{R}$  and zero. The last value, zero, is an unstable position since the stiffness is negative. Equations (1a and 1b) can be written in terms of the postbuckling displacement  $q_0$  as

$$q^3 - q_0^2 q = p \quad (2a)$$

$$\ddot{q}/\Omega^2 + 2\zeta\dot{q}/\Omega + q^3 - q_0^2 q = p \quad (2b)$$

### Equations for Predicting Nonlinear Random Responses

Since the mean values of snap-through motion changes with increasing excitation level, the technique of equivalent linearization is used to evaluate the mean value as well as the mean square value of the random response. The procedure is similar to that used for a problem of random vibration of cylindrical plates (ref. 15).

Substituting  $q = \bar{q} + \Delta q$  where  $\bar{q}$  is the mean value and  $\Delta q$  is the oscillating component and noting that  $\overline{\Delta q} = 0$ , it can be shown that

$$\begin{aligned} \overline{q^3} &= \overline{(\bar{q} + \Delta q)^3} = \bar{q}^3 + 3 \overline{\bar{q}^2 \Delta q} + 3 \overline{\bar{q} (\Delta q)^2} + \overline{(\Delta q)^3} \\ &= \bar{q}^3 + 3 \bar{q} \overline{(\Delta q)^2} \end{aligned} \quad (3)$$

Since  $\bar{P} = 0$ , and using the value of  $\overline{q^3}$  from equation (3) the mean of the equation of motion (2a) is given by

$$\bar{q}^3 + 3 \bar{q} \overline{(\Delta q)^2} - q_0^2 \bar{q} = 0$$

The solutions are

$$\bar{q}^2 = q_0^2 - 3 \overline{(\Delta q)^2} \text{ and } \bar{q} = 0 \quad (4)$$

Using equivalent linearization, the dynamic equation (2b) can be written as

$$\Delta \ddot{q} + 2\Omega\zeta\Delta \dot{q} + \Omega^2 (\overline{3q^2} - q_0^2)\Delta q = \Omega^2 p \quad (5)$$

Substituting  $\overline{q^2} = \overline{\bar{q}}^2 + (\Delta q)^2$  and  $\overline{\bar{q}}^2 = q_0^2 - 3(\Delta q)^2$  from equation (4) into (5), the dynamic equation becomes,

$$\Delta \ddot{q} + 2\Omega\zeta\Delta \dot{q} + \Omega^2 [2q_0^2 - 6(\Delta q)^2] \Delta q = \Omega^2 p \quad (6)$$

From equation (6), the random response is given by (See Appendix II of ref. 10)

$$(\Delta q)^2 = \frac{\alpha}{2[q_0^2 - 3(\Delta q)^2]} \quad (7)$$

where  $\alpha$  is a nondimensional excitation parameter given by

$$\alpha = \frac{S_{pp}(f)}{S_{pp}(f_0)_L}$$

$S_{pp}(f)$  is the spectral density of  $p$  at frequency  $f$  and  $S_{pp}(f)_L$  is the spectral density of  $p$  when the r.m.s. value of  $q$  equals 1. Solving equation 7

$$(\Delta q)^2 = \left[ \frac{1 - \sqrt{1 - 6\alpha/q_0^4}}{6} \right] q_0^2 \quad (8)$$

Substituting  $\overline{q^2} = \overline{\bar{q}}^2 + (\Delta q)^2$  and  $\overline{\bar{q}}^2 = 0$  from equation 4 into equation 5, the dynamic equation becomes

$$\Delta \ddot{q} + 2\Omega\zeta\Delta \dot{q} + \Omega^2 [3(\Delta q)^2 - q_0^2]\Delta q = \Omega^2 p \quad (9)$$

From equation 9, the random response is given by (See Appendix II, of ref. 10)

$$(\Delta q^2) = \frac{\alpha}{3(\Delta q)^2 - q_0^2} \quad (10)$$

Solving equation 10

$$\overline{\Delta q^2} = \left[ \frac{1 + \sqrt{1 + 12\alpha/(q_0^4)}}{6} \right] q_0^2 \quad (11)$$

The relationship between dynamic response and excitation level is shown in figure 2. It can be seen that with increasing excitation the random response would follow the first solution until it reaches A, where  $\alpha = q_0^4/6$ ,  $(\Delta q)^2 = q_0^2/6$ , where it would jump to the second solution due to the onset of snap-through motion. The region between B and C on the second solution curve is for intermittent snap-through motions. Point C, where  $\alpha = 2 q_0^4$  and  $(\Delta q)^2 = q_0^2$ , corresponds to the starting point of continuous snap-through motion as found by previous experimental work. It should be noted that prediction for points near A and B are subject to uncertainty because of the unsteady nature of the intermittent snap-through motion. Figure 3 indicates intermittent and continuous snap-through boundaries and values of  $(\Delta q)^2$  for various values of  $\alpha$  and  $q_0^2$ .

### APPLICATION OF SNAP-THROUGH MODEL

In order to use equations 8 and 11 to predict the dynamic response of plates it is first necessary to evaluate  $\alpha$ ,  $q_0$  for different plate parameters and boundary conditions using the procedures described below.

#### Evaluation of the Non-Dimensional Displacement Parameter

The nondimensional displacement  $q = Q/Q_N$ , where  $Q$  is the modal displacement and  $Q_N$  is the modal displacement for which the linear restoring force due to bending is equal to the nonlinear restoring force due to in-plane stretching. The nonlinear effect is significant when  $Q > 0.3 Q_N$  or  $q > 0.3$ . Values of  $(Q_N/t)^2$  for isotropic plates are shown in table 2 for different transverse and in-plane boundary conditions, as a function of aspect ratio,  $s$ , poisson ratio,  $\nu$ , and in-plane stiffness parameter,  $\gamma$ . It should be noted that  $(Q_N/t)^2$  is independent of the dimensions and elastic modulus of the plates. The steps of derivation for table 2 is found in reference

10 and it agrees with other results (ref. 1 and pg. 229 of ref. 5). The significance of  $\gamma$  is discussed in Appendix I. Based on  $\nu = 0.3$ , a nomograph for finding  $(Q_N/t)^2$  and corresponding edge bending strain  $\epsilon_N$  for clamped plates is presented in figure 4.

### The Evaluation of the Excitation Parameter

The non-dimensional excitation parameter  $\alpha$  is given by

$$\alpha = \frac{S_{pp}(f)}{S_{pp}(f_o)L}$$

Since the non-dimensional force parameter,  $p$ , is proportional to the acoustic pressure,  $h$ ,  $\alpha$  is related to the acoustic pressure spectral density by

$$\alpha = \frac{S_{hh}(f)}{S_{hh}(f_o)L}$$

In reference 10 it can be shown that (see Appendix II)

$$S_{hh}(f_o)L = 64\pi^3 c \zeta f_o^3 \rho^2 t^2 Q_N^2$$

where  $c = 0.82$  for clamped boundaries and  $c = 0.38$  for simply-supported boundaries.

From the above equations the nomograph in figure 5 has been developed for finding  $\alpha$ . The use of this nomograph requires knowledge of the modal damping coefficient,  $\zeta$ ; plate thickness,  $t$ ; fundamental resonance frequency,  $f_o$ ; plate material density,  $\rho$ ; acoustic pressure spectral density,  $S_{hh}(f)$ ; and the ratio  $(Q_N/t)^2$ . An approximate value for  $f_o$  can be found for aluminum and titanium alloy plates from the nomograph in figure 6. This nomograph is based upon results from reference 5.  $(Q_N/t)^2$  can be found from figure 4.

### Steps in Using the Design Charts

The general use of the design charts is outlined in the following steps.

1. Find  $(Q_N/t)^2$  from figure 4. Then find the bending strain  $\epsilon_N$  corresponding to  $Q_N$ .
2. Find  $f_0$  using the figure 6.
3. Find  $\alpha$  from figure 5 using the sound spectral level in dB/Hz.
4. Find  $q_0^2$  from the compression parameter,  $R$ , or from the temperature (ref. 8). It can also be found by using  $q_0 = Q_0/Q_N$  or  $\epsilon_0/\epsilon_N$  if the static buckling displacement  $Q_0$  or bending strain  $\epsilon_0$  are known.
5. The nonlinear response  $(\Delta q)^2$  corresponding to  $q_0^2$  and  $\alpha$  can be found from figure 2 or directly calculated from equation 8 and 11.
6. The r.m.s. strain response is obtained by  $\epsilon_{rms} = \sqrt{(\Delta q)^2} \epsilon_N$ .
7. The values of  $\alpha$  corresponding to snap-through boundaries for given values of  $q_0^2$  can be found from figure 3. The value of  $q_0^2$  corresponding to maximum response  $(\Delta q)^2$  can be found if the value of  $\alpha$  is given.

The following examples are used to illustrate the steps in using the design charts.

### Example 1

The specimen is an aluminum alloy plate, 12 in. x 15 in. x 0.064 in. with all sides clamped and immovable and was tested in the Thermal Acoustic Fatigue Apparatus at the NASA Langley Research Center (ref. 16). The modal damping is assumed to be 0.02

From figure 4, using  $s = 1.33$ ,  $(Q_N/t)^2$  is found to be 1.38 for an immovable edge condition. With  $t = 0.064$  in. and  $a = 12$  in., the bending strain on the shorter edge is  $\epsilon_N = 450$  microstrain.

From figure 6, using  $a = 12$  in.,  $s = 1.33$ , and  $t = 0.064$  in.,  $f$  is found to be 128 Hz.

From figure 5, using  $f = 128$ ,  $\zeta = 0.02$ ,  $(Q_N/t)^2 = 1.38$ ,  $t = 0.064$  in.,  $S_{hh} = 121$  dB/Hz., the excitation parameter  $\alpha$  is found to be 1.0.

The comparison between predicted and experimental results are summarized in table 3. For the flat plate, the present method agrees well with a linear method (ref. 17) for low SPL but gives a better estimate of the dynamic strain for  $S_{hh}$  of 130 dB/Hz.

From table 3, the prediction is not very good for the response without snap-through for  $q_0^2=0.36$ . This error is probably due to the presence of imperfections in the plate which affects the buckling displacement. However, the predictions for continuous snap-through motion are quite good. The effects of small buckling displacement becomes less significant compared with the effects of the dynamic displacement. For  $q_0^2=1.52$ , the prediction is not very good for intermittent snap-through motion.

### Example II

For this example, the specimen and conditions are similar to Example I except that the thickness is 0.040 in.

From figure 4,  $(Q_N/t)^2 = 1.38$ . For  $t = 0.040$  in. and  $a = 12$  in. the bending strain on the shorter edge is  $\epsilon_N = 195$  microstrain.

From Figure 6,  $f = 80$  Hz.

From Figure 5,  $\alpha = 1$  when the spectral level is 107 dB/Hz.

The comparison between predicted and measured results are shown in table 4. Agreement was quite good for conditions where  $q_0^2=0, 0.2, 1.0, 2.48$  except for some intermittent snap-through. For  $q_0^2=2.98, 5.10, 6.7$ , the snap-through response starts at SPL values lower than predicted. Examination of the response spectrum shows significant resonance response of the second mode at a frequency close to the fundamental frequency. The differences between the results from the present method and the finite element method in reference 2 are less than 30 percent.

### Example III

For this example the specimen is an aluminum beam, 15 in. x 2 in. x 0.032 in.,

clamped at both ends with base excitation of uniform spectral density from 20 - 100 Hz. The modal damping,  $\zeta$ , was found to be 0.017 (details in Appendix III).

From Figure 4, using  $s = \infty$  (actually  $s=6$  is adequate) and  $S_p/S_B = 0.25$  (the clamp is movable due to bending of the base), it is found that  $(Q_N/t)^2 = 1.7$  and  $\epsilon_N = 71.8$  microstrain.

From Figure 6, using  $s = \infty$ ,  $f = 30$  Hz.

From Figure 5,  $\alpha=1$  when the spectral level is 91 dB/Hz.

The comparison in table 5 shows very good agreement for the flat beam. The large errors for the response of the buckled beam without snap-through are possibly due to imperfections. However, the responses with snap-through motion show better agreement.

### DISCUSSION

It is expected that experimental errors included both errors in damping ratio and in transverse boundary conditions. The accuracy of the prediction results depends mainly on the evaluation of the non-linear stiffness, which is greatly affected by the in-plane boundary conditions.

Near the buckling point, the displacement is greatly affected by imperfection in the plate and the two static positions may not be symmetric about the flat position. This would have significant influence on the onset of snap-through motions and on the non-linear response without snap-through motions. Consequently, the errors in response without snap-through were generally greater than errors in response with continuous snap-through. It should be noted that intermittent snap-through motions are of small magnitude and excited by low excitation level, thus they are generally

not as important for fatigue considerations. The continuous snap-through motions are more important and can be predicted reasonably well by equation 11.

For high buckling displacement ( $q_0^2 > 2.5$ ) there is a possibility of coupling between the first two modes in the dynamic response and, therefore, snap-through motion may initiate at an excitation level lower than predicted. The (1,1), (2,1) modes are found to be coupled during static snap-through (ref. 13).

For the three examples discussed, the errors of the predicted results are mostly within  $\pm 30$  percent of the experimental results for  $q_0^2 \leq 2.5$  and  $\alpha \leq 35$ . These ranges of buckling and excitation parameters are adequate for most practical purposes. To improve the design charts, equation 8 and 11 should be derived with consideration of higher modes.

### CONCLUDING REMARKS

A simple and direct design procedure for estimating the nonlinear response of flat and buckled plates has been developed. The procedure gives good results for moderately buckled plates under moderate excitation. The major source of error for highly buckled plates is attributed to the coupling between the first two modes of vibration. Other sources of error include imperfection in the plates, damping estimates and boundary conditions. For most practical structures, the range of application of the design procedure is adequate for first order estimation of the nonlinear response. However, the most important step is to evaluate correctly the in-plane boundary condition.

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## APPENDIX I

### The Influence of In-Plane Boundary Condition

From reference 10, the in-plane stresses  $\sigma_x$ ,  $\sigma_y$  are given by

$$\sigma_x = \sigma_{xm} + \sigma_{xc}$$

$$\sigma_y = \sigma_{ym} + \sigma_{yc} \quad (A1)$$

where  $\sigma_{xm}$ ,  $\sigma_{ym}$  are the mean of  $\sigma_x$ ,  $\sigma_y$  respectively over the area of the plate,

$\sigma_{xc}$ ,  $\sigma_{yc}$  are the corresponding varying components.

The strain energy is given by equation A9 of reference 10:

$$V_e = E t a b Q^4 + \frac{t a b}{E} (\sigma_{xm}^2 + \sigma_{ym}^2 - 2\nu \sigma_{xm} \sigma_{ym}) \quad (A2)$$

where  $e$  = a constant related to the total strain energy due to  $\sigma_{xc}$ ,  $\sigma_{yc}$ .

(see reference 10 or reference 1)

It should be noted that  $e$  depends only on the bending magnitude of the plate and is independent of the in-plane boundary condition.  $\sigma_{xm}$ ,  $\sigma_{ym}$  are dependent on the boundary conditions and are given by equation A6 of reference 10.

$$\begin{aligned} \left(1 + \frac{S_P}{S_B}\right) \sigma_{xm} &= \frac{E}{(1 - \nu^2)} (H_x + \nu H_y) Q^2 \\ \left(1 + \frac{S_P}{S_B}\right) \sigma_{ym} &= \frac{E}{(1 - \nu^2)} (H_y + \nu H_x) Q^2 \end{aligned} \quad (A3)$$

where  $S_P$ ,  $S_B$  = in-plane stiffness of the plate and boundary respectively.

$$H_x = \frac{1}{2ab} \int_0^b \int_0^a \left(\frac{\partial \phi}{\partial x}\right)^2 dx dy$$

$$H_y = \frac{1}{2ab} \int_0^b \int_0^a \left(\frac{\partial \phi}{\partial y}\right)^2 dx dy$$

The influence of in-plane boundary condition is not considered in reference 1 and 5.

Note that nonlinear restoring force  $= K_N Q^3 = \partial V_e / \partial Q$ .

## APPENDIX II

### Derivation of Reference Sound Spectral Level $S_{pp}(f_o)_L$ .

From reference 10

$$S_{pp}(f_o)_L = \frac{\delta \zeta}{\Omega} = \frac{4\zeta}{\pi f_o} \quad (B1)$$

$$p = \frac{P}{KQ_N} \quad (B2)$$

$$P = h I_1 \text{ where } I_1 = \int_0^b \int_0^a \phi(x, y) dx dy \quad (B3)$$

$$K = M \Omega_o^2 = M(2\pi f_o)^2 \quad (B4)$$

$$M = \text{modal mass} = \rho t I_2$$

$$= \text{where } I_2 = \int_0^b \int_0^a \phi^2(x, y) dx dy$$

$$\text{Thus } K = 4\pi^2 f_o^2 \rho t I_2 \quad (B5)$$

Substituting P from equation B3, K from equation B6 into equation B2

$$p = \frac{h I_1}{4\pi^2 f_o^2 \rho t I_2 Q_N} \text{ or } h = 4\pi^2 f_o^2 \rho t Q_N \left( \frac{I_2}{I_1} \right) p$$

Therefore,

$$S_{hh}(f_o)_L = 16\pi^4 f_o^4 \rho^2 t^2 Q_N^2 \left( \frac{I_2}{I_1} \right)^2 S_{pp}(f_o)_L$$

$$= 64\pi^3 c \zeta f_o^3 \rho^2 t^2 Q_N^2 \quad (\text{from B1}) \quad (B7)$$

$$c = \left( \frac{I_2}{I_1} \right)^2$$

### APPENDIX III

#### Base Excitation Test

Base excitation involve the excitation of the supporting frame of the specimen by electromagnetic shaker. The specimen will experience an equivalent uniform pressure excitation proportional to the acceleration of the base and is given by (Derivation in ref. 18).

$$h = A \rho t$$

where  $A$  is the base acceleration in  $\text{in}/\text{sec}^2$ ,  $\rho$  is density of the beam in  $\text{lbm}/\text{in}^3$ ,  $t$  is the thickness of the beam in inches.

$$\text{Thus } S_{hh}(f) = \rho^2 t^2 S_{AA}(f)$$

The spectrum of base acceleration  $S_{AA}(f)$  is found to be quite flat above 20 Hz.

Tests with sinusoidal excitation are performed initially. It is found that the use of single mode can predict the fundamental frequency, displacement response (measured by a small accelerometer at the center of the beam), strain response (at the edge) to within 10 percent of the measured values.

**Table 1. Comparison of the Effects of Intermittent and Continuous Snap-Through Motion**

DISPLACEMENT OR STRAIN RESPONSE	SNAP-THROUGH MOTION		
	NONE	INTERMITTENT	CONTINUOUS
MEAN	STATIC VALUE	UNSTEADY	ZERO
R.M.S	< 20% STATIC VALUE	JUMPS WITH SPL	> 70% OF STATIC VALUE
SPECTRUM	MODAL	PEAK AT 0 HZ	MODAL WITH PEAK BROADENING
NONLINEARITY	SOFTENING	UNSTABLE	HARDENING

**Table 2. Nonlinear Reference Displacement  $(Q_N/t)^2$**

$S_P$  = in-plane stiffness of the plate

$S_B$  = in-plane stiffness of the boundary

$s$  = aspect ratio

$$A = 1/s + s$$

$$B = A^2 - 2$$

$$\gamma = \left(1 + \frac{S_P}{S_B}\right)^{-1}$$

Transverse In-Plane	Simply-Supported	Clamped
Movable (edges straight) $S_P / S_B = \infty$	$\frac{4}{3(1-\nu^2)} \frac{A^2}{B}$	$\frac{8(B + 0.66)}{(1-\nu^2)(\frac{17}{8}B + 4A^2)}$
Im movable $S_P / S_B = 0$	$\frac{4}{3} \frac{A^2}{[4(B + \nu) + B(1-\nu^2)]}$	$\frac{8(B + 0.66)}{(1-\nu^2)[4.5(B + 2\nu) + (\frac{17}{8}B + 4A^2)]}$
Partially Movable	$\frac{4}{3} \frac{A^2}{[4\gamma(B + \nu) + B(1-\nu^2)]}$	$\frac{8(B + 0.66)}{(1-\nu^2)[4.5\gamma(B + 2\nu) + (\frac{17}{8}B + 4A^2)]}$



**Table 3. R.M.S. Strain Response of an Aluminum Plate (12 in. x 15 in. x 0.064 in.) to Broadband Excitation**

( ) result from reference 17 using linear theory

Buckling Condition	Spectral Level $S_{hh}$ (dB/Hz)	Predicted $\epsilon_{rms}$ (microstrain)	Measured $\epsilon_{rms}$ (microstrain)
flat $q_0^2 = 0.0$ $\epsilon_N = 450$ $S_L = 121$ dB/Hz	100	35 (30)	41
	110	115 (95)	112
	120	275 (300)	262
	130	545 (950)	443
buckled $q_0^2 = 0.36$ $\epsilon_0 = 270$ $S_S = 104.2$ dB/Hz $S_C = 115$ dB/Hz	110	218 st	85
	115	270	173 st
	120	344	307
	122	382	396
	124	424	487
	130	589	573
buckled $q_0^2 = 1.52$ $\epsilon_0 = 553.4$ $S_S = 116.8$ dB/Hz $S_C = 127.6$ dB/Hz	105	41	31
	110	75	81
	115	146	133
	125	493 st	360 st
	130	620	556

st = start of snap-through

**Table 4. R.M.S. Strain Response of an Aluminum Plate (12 in. x 15 in. x 0.40 in.) to Broadband Excitation**

( ) result from reference 2 using finite element method

Buckling Condition	Spectral Level $S_{hh}$ (dB/Hz)	Predicted $\epsilon_{rms}$ (microstrain)	Measured $\epsilon_{rms}$ (microstrain)
a) Flat plate $\epsilon_N = 195$ $S_L = 107$ dB/Hz	110 115 120 125	159 (190) 222 303 (405) 410	169 184 265 382
buckled $q_0^2 = 0.2$ $\epsilon_0 = 87.2$ $S_S = 85.3$ dB/Hz $S_C = 96.1$ dB/Hz	110 115 120 125	179 (210) st 236 314 (410) 417	252 st 326 414 500
buckled $q_0^2 = 1.0$ $\epsilon_0 = 195$ $S_S = 99.2$ dB/Hz $S_C = 110$ dB/Hz	110 118 123 128	195 (210) st 291 381 (440) 503	160 206 470 564
buckled $q_0^2 = 2.48$ $\epsilon_0 = 307$ $S_S = 107$ dB/Hz $S_C = 117.8$ dB/Hz	105 110 122 127	78 225 st 376 488	89 165 st 574 638
buckled $q_0^2 = 2.98$ $\epsilon_0 = 336.6$ $S_S = 108.7$ dB/Hz $S_C = 119.5$ dB/Hz	102 106 122 127	46 78 379 st 489	89 165 st 470 694
buckled $q_0^2 = 5.10$ $\epsilon_0 = 440$ $S_S = 113.2$ dB/Hz $S_C = 124.0$ dB/Hz	101 105 109 119	31 53 83 356 st	158 202 st 398 676
buckled $q_0^2 = 6.7$ $\epsilon_0 = 505$ $S_S = 115.8$ dB/Hz $S_C = 126.6$ dB/Hz	101 105 110 115	26 42 78 158	78 140 276 st 456

st = start of snap-through

**Table 5. R.M.S. Strain Response of an Aluminum Beam (2 in. x 15 in. x 0.032 in.) to Broadband Excitation**

Buckling Condition	Spectral Level $S_{hh}$ (dB/Hz)	Predicted $\epsilon_{rms}$ (microstrain)	Measured $\epsilon_{rms}$ (microstrain)
flat $\epsilon_N = 71.8$ $S_L = 91$ dB/Hz	65	23.6	2.4
	76.5	13.0	9.2
	83.4	25.5	20.2
	91.7	49.8	41.2
	95.9	71.1	61.0
	102.3	100.5	94.0
	103.9	111.0	101.5
	105.5	122.3	116.0
	108.1	143.1	131.0
	109.8	158.4	143.5
Buckled $q_o^2 = 3.64$ $\epsilon_o = 161.3$ $S_S = 94.2$ dB/Hz $S_C = 105$ dB/Hz	79	8.1	2.2
	88.7	25.7	7.4
	94.1	60.8	18.9
	95.3	111.9 st	33.3 st
	96.3	115.2	46.8
	103.6	151.4	114.5
	106.4	172.2	148.5
	108.0	186.0	162.5
	109.4	199.6	173.0

st = start of snap-through

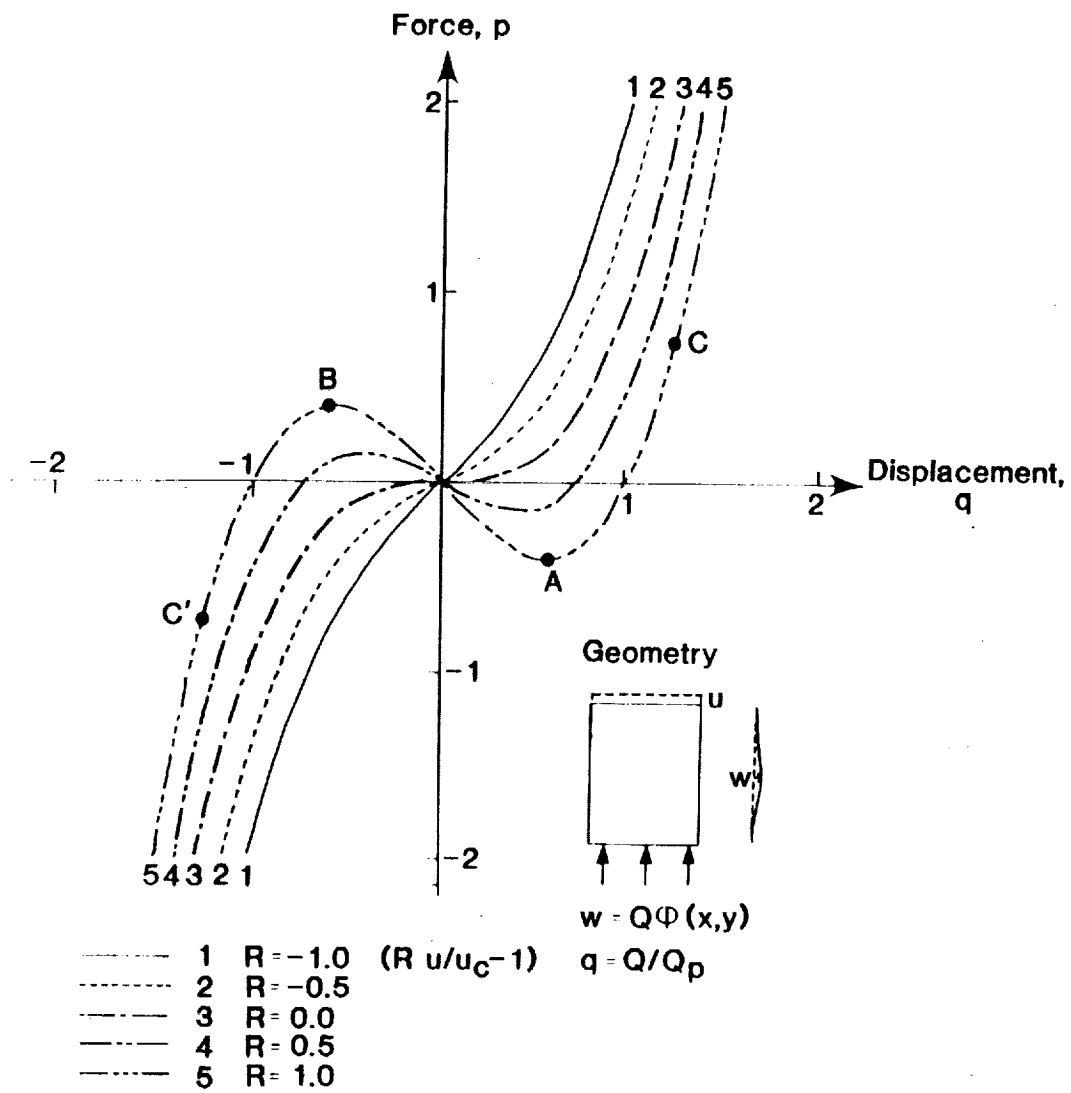


Figure 1. Relationship between nondimensional displacement and force.

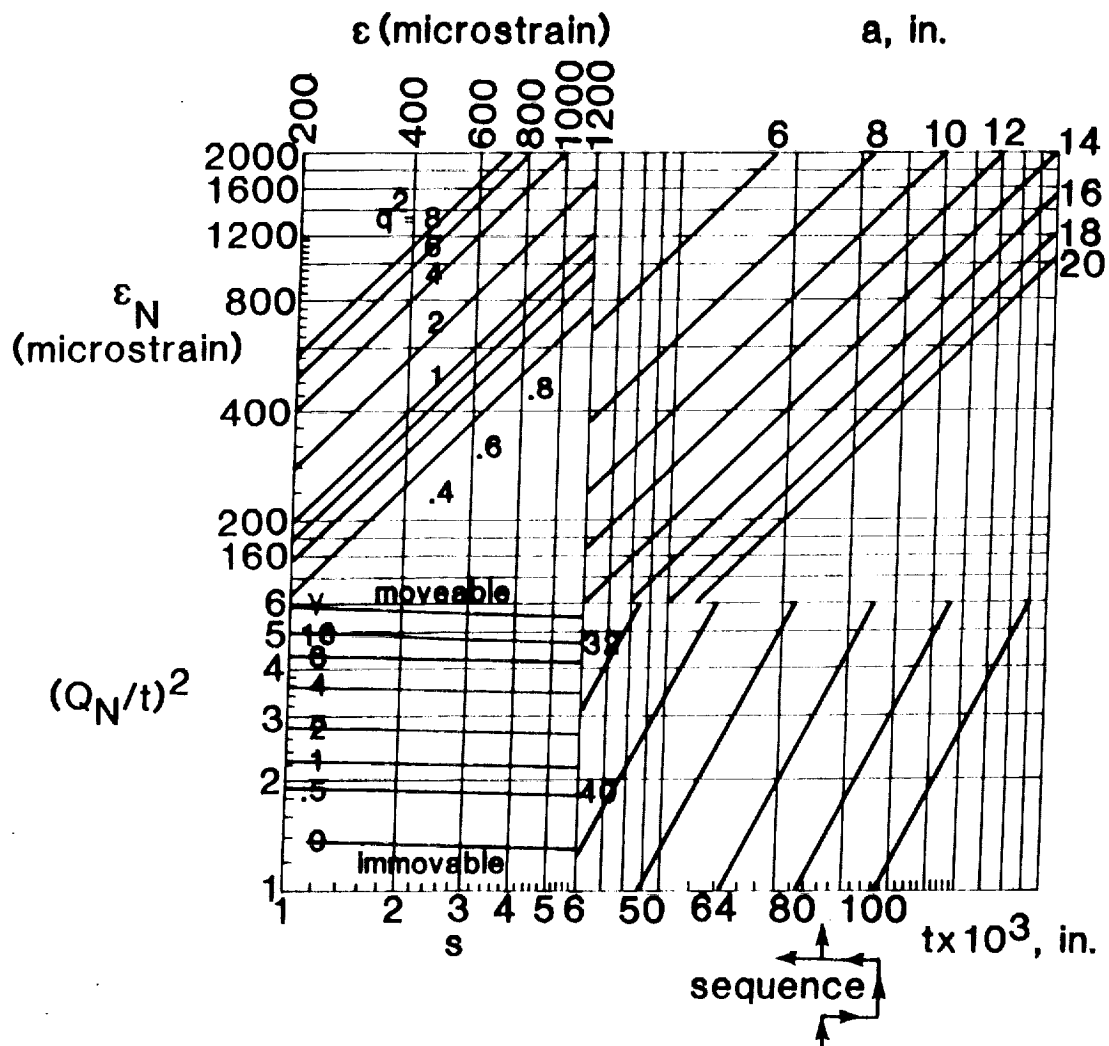


Figure 2. Relationship between random response and excitation level.

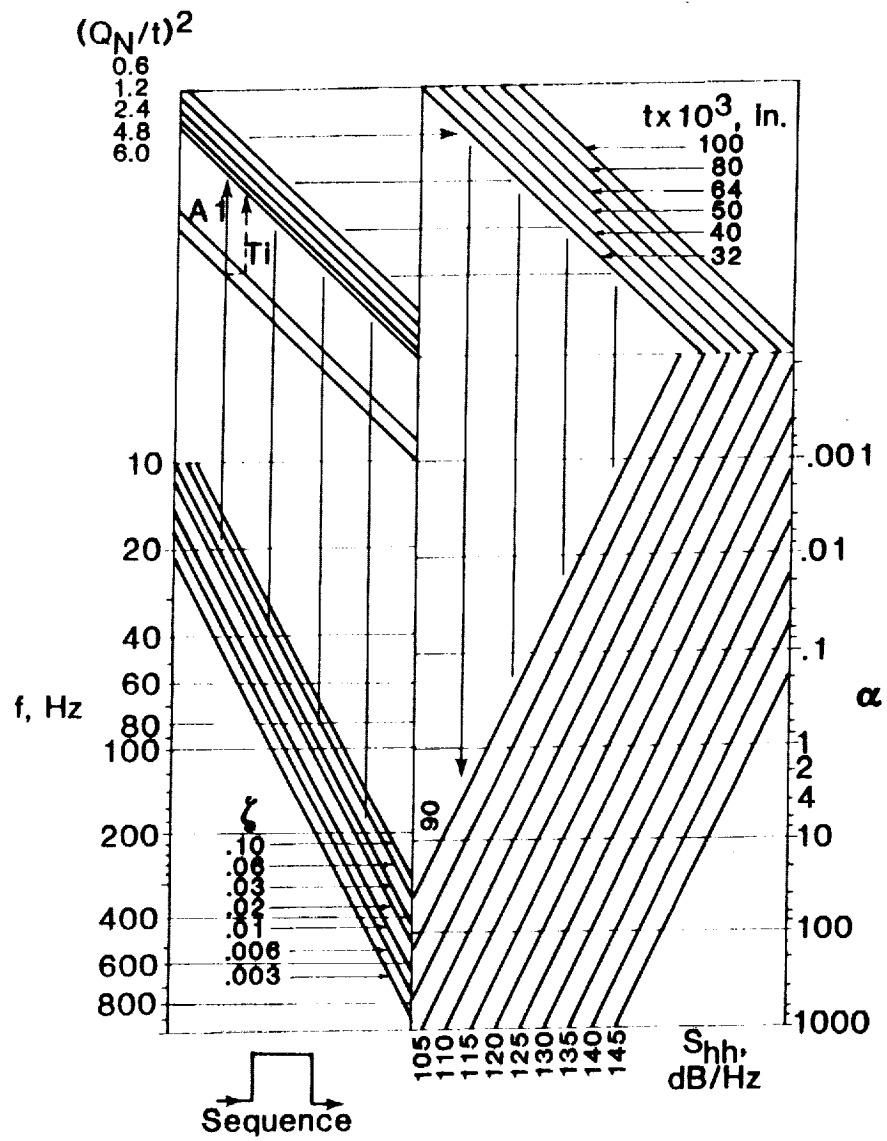


Figure 3. Intermittent and continuous snap-through boundaries and relationship

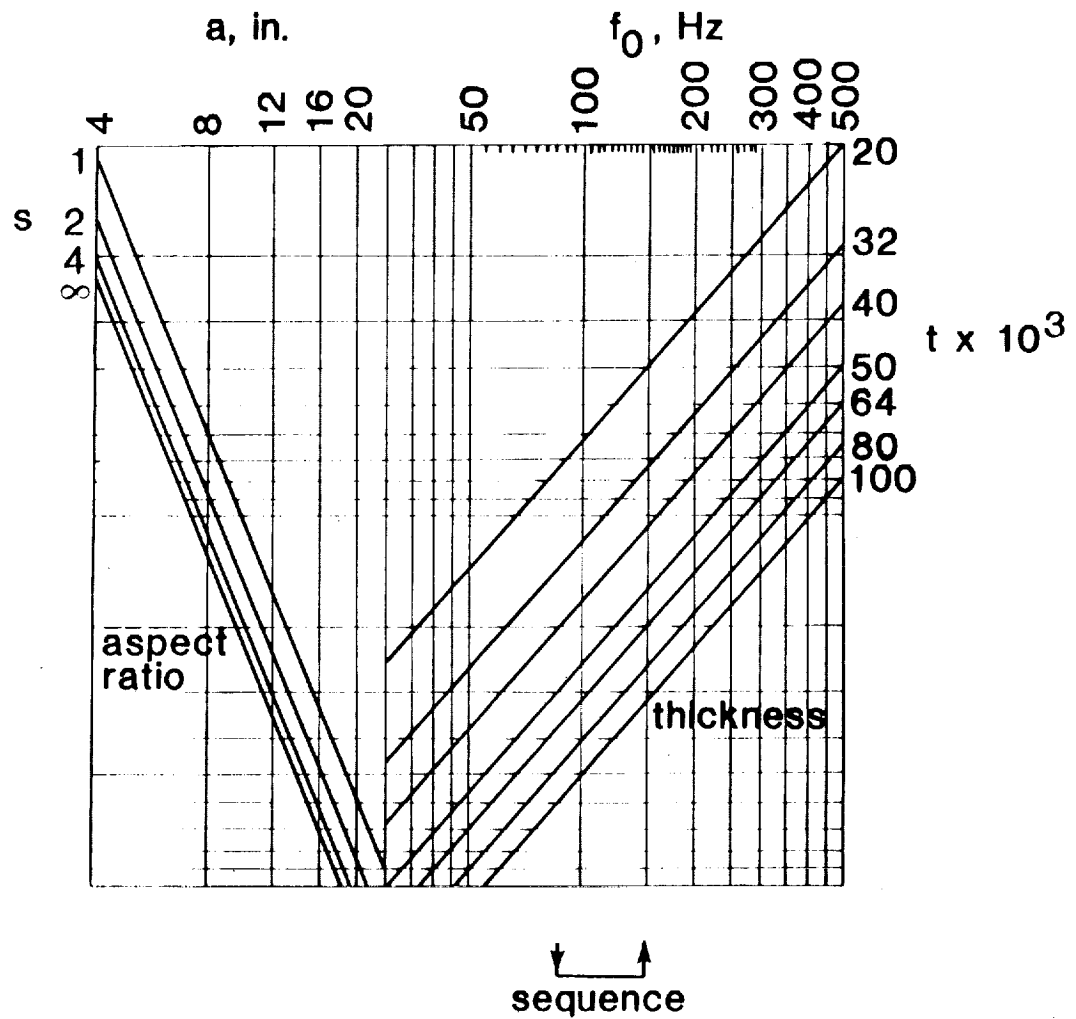


Figure 4. Nomograph for evaluating modal displacement,  $Q_N$ , and surface strain,  $\epsilon_N$ , when the linear restoring force equals the nonlinear force.

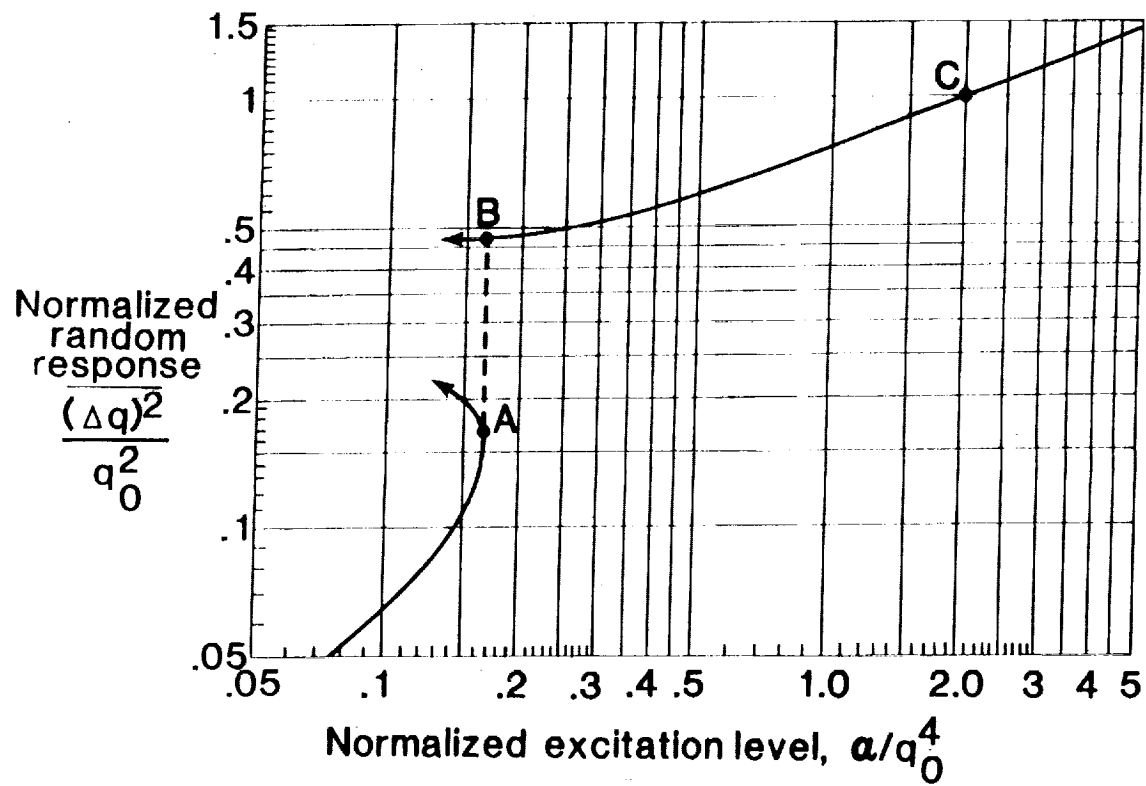


Figure 5. Nomograph for evaluating the nondimensional excitation parameter,  $\alpha$ .



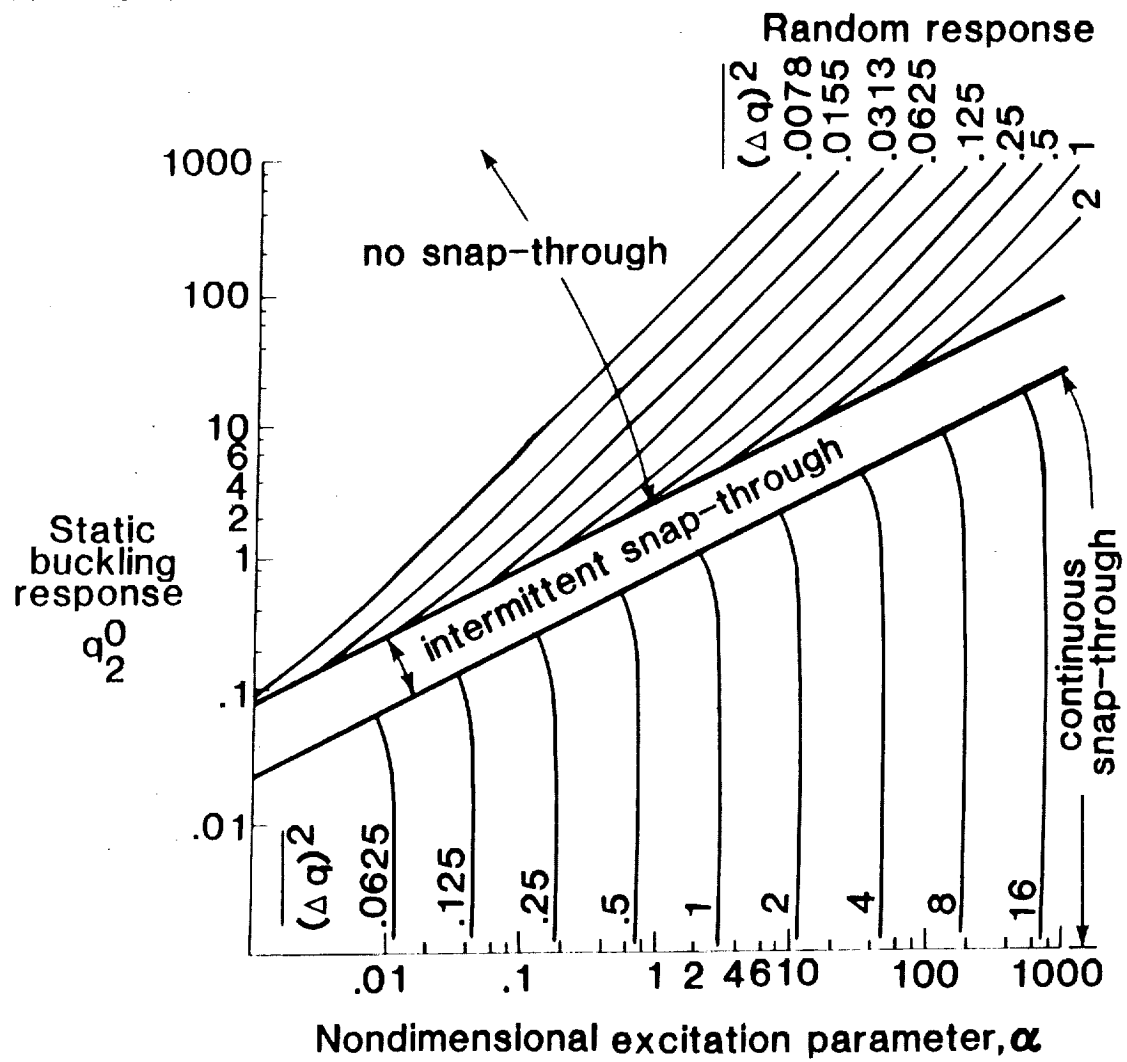


Figure 6. Nomograph for evaluating fundamental frequency,  $f_0$ .



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